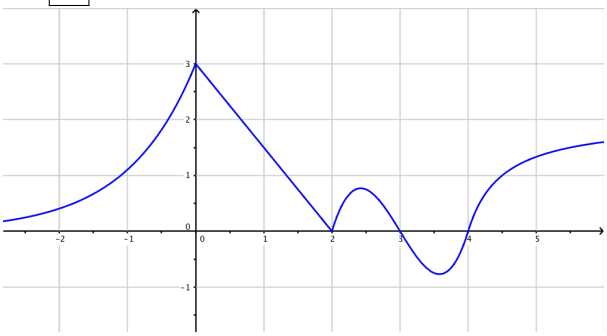
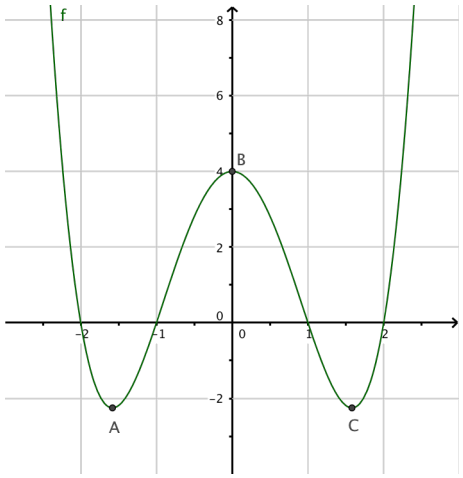
LECTURE NOTES: 4-1 MAXIMUM AND MINIMUM VALUES (PART 1)

MOTIVATING EXAMPLES

A



B



DEFINITIONS: Let $f(x)$ be a function with domain D and let c be an x-value in D. Then the y-value $f(c)$ is:

1. **an absolute maximum** if

2. **an absolute minimum** if

3. **a local maximum** if

4. **a local minimum** if

ARE WE ALL ON THE SAME PAGE?

1. What sort of *category* is a maximum (or minimum)? (Animal, vegetable, number, point, x-value, y-value, mineral...?)

2. Can function have more than ONE maximum (or minimum)?

3. Can a function have neither a maximum nor a minimum?

4. Looking at our earlier pictures, at what sort of places do maximums and minimums appear?

POWERFUL TOOL #1: The Extreme Value Theorem
Given a function $f(x)$ such that

1.

2.

then $f(x)$ is *guaranteed* to have a maximum and a minimum _____.

PRACTICE PROBLEMS: For each function with designated region, sketch the graph to determine its absolute maximum and its absolute minimum, if they exist. If they do not exist, explain why the Extreme Value Theorem is not violated.

1. $f(x) = 1 - x^2$ on $[-1, 1]$

2. $g(x) = \frac{-2}{x-1}$ on $[0, 2]$

3. $h(x) = \cos x$ on $(0, \pi)$

4. $k(x) = (x + 1)^3$ on $[-2, 3]$

POWERFUL TOOL #2: Critical Points

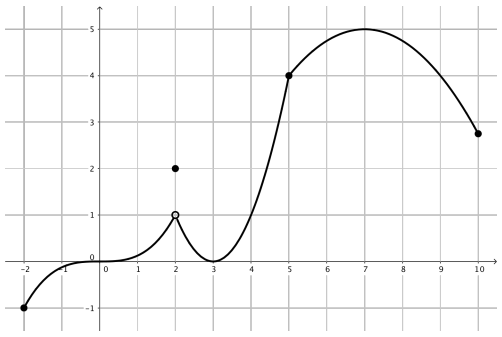
Definition: A **critical number** of a function $f(x)$ is an ____-value c _____ such that either

(a) or (b) .

Why do we care about critical points?

MORE PRACTICE PROBLEMS: For each function below, (a) find all critical points (b) identify all local maximums and local minimums (if any) and *where they occur* (c) identify all absolute maximum and minimums and *where they occur*. If no domain is explicitly stated, assume you are using the natural domain of the function as written. **You are expected to provide clear coherent explanations of how you deduced your answers.**

1. $f(x)$ is graphed below:



2. $f(x) = \begin{cases} 2x+1 & \text{if} \quad 0 \le x < 1 \\ 4-2x & \text{if} \quad 1 \le x \le 3 \end{cases}$

3. $g(x) = \ln x$ on $(0, 2]$

4. $f(t) = t^4 + t^3 + t^2 + 1$

5. $h(x) = x^{4/5}(x-4)^2$ (You will probably need to use a calculator to graph this one.)

6. $f(x) = 5 + 54x - 2x^3$ on $[0, 4]$

7. $g(x) = x + \frac{1}{x}$ on $[0.2, 4]$